The use of a hybrid method combining finite element calculation of the parameters in the channel cross section (the use of planar elements) and finite-difference approximation lengthwise is evidently the most effective and expedient approach with regard to saving computer storage and cutting processor operating time. The latter is 30-40 sec for one step on an ÉS-1061 computer.

NOTATION

U, velocity; T, temperature; C_k, concentration of the k-th component; x, y, z, cartesian coordinate system; ρ, density; λ_f, frozen thermal conductivity; μ_t, λ_t, and D_t, turbulent values of the viscosity coefficient, thermal conductivity, and diffusion, respectively; $\overline{Nu}$, mean Nusselt number for both sides T_1 and T_2. Indices: (e) – for an element; wa – wall.

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APPROXIMATE SOLUTION OF A PROBLEM OF CONVECTIVE HEAT TRANSFER BETWEEN A PLATE AND LIQUID METALS

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UDC 532.526.4:536.242

This article examines a theoretical method of calculating the heat-transfer coefficient for different values of the Reynolds number of a liquid-metal flow onto a plate.

The differential (local) method has become the method most commonly used in the general theory of qualitative and quantitative description of heat transfer under conditions of wall turbulence. In this method, turbulent heat transfer is completely determined by the physical parameters (density, viscosity, distributions of mean velocities and temperature) of a uniform fluid flow (liquid metals, gas, liquids in drop form) [1]. If we connect a translating coordinate system with a local fluid particle, then in accordance with Galileo's principle all of the dynamic processes of turbulent transport will occur identically in regard to this inertial system of reference [1].

Let only two physical quantities – momentum and heat – be transported through streamlines representing the averaged motion of the fluid medium. Then the transfer of momentum creates turbulent friction between the layers of the fluid, while heat transfer results in turbulent heat conduction. Since there are no other factors contributing to turbulent heat transfer in the given case, the turbulent mixing mechanism will be the same for both turbulent friction and turbulent heat conduction [1]. Meanwhile, the same volumes of fluid simultaneously transfer momentum and heat. If no heat is exchanged with the environment, then it follows from the Prandtl theory [1] that if momentum is conserved, then the amount of heat transferred by the fluid volumes is also conserved. This leads to a situation whereby the turbulent Prandtl number, characterizing the connection between turbulent transfer of

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 2, pp. 253-258, August, 1989. Original article submitted February 23, 1988.

momentum and heat, is equal to unity [1]. Also, the thicknesses of the hydrodynamic and thermal turbulent boundary layers (TBL) can be assumed to coincide in this instance. Thus, in the framework of the given formulation of the problem, the main issue remaining unresolved with regard to heat transfer under conditions of wall turbulence is the determination of the shear stress associated with turbulent friction.

Following Boussinesq's notion [1], we can determine turbulent friction as a function of the nonuniformity of the field of averaged velocities near each point of the flow. Then assuming the shear stress to consist of viscous and turbulent components and having written Fourier's law in the form of its turbulent analog, we can use the Newton–Reichmann law in the example of a hydraulically smooth plate of length L to calculate the heat transfer from a solid to a flow of a uniform one-phase fluid washing over it.

Here, to ensure intensive heat removal from the heating surfaces of differential structural elements of power-plant equipment, we examined liquid metals as the uniform fluid. Liquid metals have a higher thermal conductivity than liquids in drop form or gases [1-4].

Let the velocity u_∞ of the liquid-metal flow incident on the plate be such that $\mathrm{Re} \gg 1$, while the thickness of the TBL $\delta/L \ll 1$. In this case, the TBL on the plate can be considered planar [5]. Following Prandtl [1], pulsations of velocity normal to the streamlines representing averaged motion can be assumed to be proportional to the difference between the velocities of the fluid layers. The axis of the coordinate system is directed perpendicular to the streamlines. Then the basic equation of heat conduction under wall-turbulence conditions [1-5] has the following form [6] in dimensionless variables after some simple transformations:

$$\nu = (\nu + \varepsilon_s) \frac{dW}{d\eta}, \tag{1}$$

$$Q = \left(\frac{\varepsilon_s}{\mathrm{Pr}_t} + \frac{\nu}{\mathrm{Pr}} \right) \frac{W_k}{\nu} \frac{dt}{d\eta}. \tag{2}$$

If the shear stress and heat flux in the TBL do not change along y and the temperature on the plate surface t_c is fixed, then with knowledge of the turbulent friction law and integration of Eqs. (1) and (2) over the thickness of the TBL, we can determine the temperature gradient in the TBL. However, no such solution can be obtained because the friction law is unknown in the region of contact of the thermal sublayer with the turbulent core in a TBL [5]. Various simplifications have been made in the literature [1, 3-6] to calculate heat transfer on the basis of Eqs. (1) and (2). For example, investigators have used two-layer schemes [1, 3-5] for a thermal TBL (without allowance for the region of contact of the thermal sublayer with the turbulent core) and three-layer TBL models [1, 6] in which the unknown friction law is approximated by a specially chosen function. For these reasons, the solutions of Eqs. (1)-(2) presented in [1, 3-6] have a very limited range of application. For example, the solutions based on two-layer schemes for the TBL are valid only for $\mathrm{Pr} \approx 1$ [4], while the solutions based on three-layer TBL models are valid for $1 \le \mathrm{Pr} < 10$ [1]. Thus, the main method of investigation is experimentation [2-4]. However, the empirical solutions in [2-4] are valid within the narrow ranges in which they were substantiated experimentally. Thus, it is important to continue the search for methods of calculating heat transfer under conditions of wall turbulence.

Here, our theoretical solution is based on the three-layer Kármán model of a thermal TBL. The TBL consists of a thermal sublayer, the region of contact of this sublayer with the turbulent core (buffer layer), and the turbulent core [1]. We used the following assumptions: 1) the structure of the flow in each sublayer of a thermal TBL is strictly determined; 2) molecular heat conduction predominates over turbulent heat transfer in the thermal sublayer; 3) turbulent heat transfer occurs along with molecular heat conduction in the buffer layer, but molecular conduction is the decisive factor here; 4) turbulent heat transfer predominates over molecular heat conduction in the turbulent core of the TBL. The problem in [7, 8] is taken as the basis for calculating shear stress in the buffer layer of the TBL, this approach representing a continuation of the solution for the friction law from the thermal sublayer to the turbulent core of the TBL. Thus, within the framework of the above assumptions for the problem of calculating heat transfer between a plate and a flow of liquid metal, the temperature gradient in the thermal TBL is composed of the sum of the temperature gradients in each sublayer determined by integrating (2) over the thickness of the TBL with a known shear stress. The thickness of the TBL is found by solving a boundary-value problem obtained on the basis of the impulse theorem [5]:

$$\delta_t = \mathrm{Re}\,(0{,}5c_f)^{-\frac{3}{2}}\,(n+1)\,(n+2)/n. \quad (3)$$

On the basis of the empirical formula in [3, 4] for the relationship between the thermal and viscous sublayers of a hydrodynamic TBL, we determine the thickness k_1 of the thermal sublayer:

$$k_1 = \frac{\eta_1}{\sqrt[3]{\mathrm{Pr}}}. \quad (4)$$

The thickness η_1 of the viscous sublayer of the hydrodynamic TBL is determined by solving the equation for the universal profile of the velocity of the boundary layer [1, 3] after we insert into it the value of δ_t from (3):

$$\eta_1 - \frac{1}{\varkappa}\ln\eta_1 = \sqrt{\frac{2}{c_f}} - \frac{1}{\varkappa}\ln\delta_t.$$

In the thermal sublayer $[0, k_1]$ the velocity profile can be approximated by the following linear function on the basis of the experimental data in [1, 3-5]:

$$W(\eta) = k\eta, \quad (5)$$

where $k = 1$ for heat transfer between the plate and the flow of viscous incompressible fluid washing over it ($\mathrm{Pr} \geq 1$).

In the case of heat transfer between the plate and a flow of liquid metal washing over it ($\mathrm{Pr} \ll 1$), the slope in (5) can be determined through the ratio

$$k = \frac{W(k_1)}{k_1}, \quad (6)$$

where $W(k_1) = \eta_1$ is the velocity of the thermal TBL at the point $\eta = k_1$.

We use the solution of Eqs. (1)-(2) to find the temperature gradient in the thermal sublayer $[0, k_1]$:

$$\Delta t_1 = \frac{Q}{W_k} A\eta_1 \mathrm{Pr}^{2/3}, \quad (7)$$

where

$$A = \frac{k\mathrm{Pr}_t}{k\,\mathrm{Pr}_t + \mathrm{Pr}\,(1-k)}.$$

According to the experimental data in [4], the liquid metal undergoes slight pulsative motion in the buffer layer. Then, considering the high molecular thermal conductivity of liquid metals, we can use the following empirical relation [1] for ε_s in the buffer layer

$$\varepsilon_s = \nu\gamma\varkappa^4\eta^4, \quad (8)$$

this equation being valid for regions of the TBL where molecular diffusion predominates over the pulsative motion of the uniform fluid. Then we obtain the upper boundary k_2 of the buffer layer from the solution of the corresponding boundary-value problem obtained on the basis of momentum transport equation (1). After some simple transformations, we find the upper boundary of the buffer layer k_2 from the relation

$$\frac{1+(k_1-k_0)^2}{1+\gamma\varkappa^4k_1^4}\,\mathrm{arctg}\,\frac{k_0-k_1}{1+k_2k_1-k_0(k_2-k_1)+k_0^2} + \eta_1 - B(k_2-\eta_1)^{\frac{1}{m}} = 0, \quad (9)$$

where

$$k_0 = [k_1 - hk_2 + \sqrt{(k_1-k_2h)^2 - a_1a_2}]/(1-h);\quad a_1 = 1-h;$$

$$B = \left(\sqrt{\frac{2}{c_f}} - \eta_1\right)\Big/ (\delta_t - \eta_1)^{\frac{1}{m}};\quad a_2 = 1 + k_1^2 - h(1+k_2^2);$$

$$h = \frac{B}{m}(1+\gamma\varkappa^4k_1^4)\,(k_2-\eta_1)^{\frac{1}{m}-1}.$$

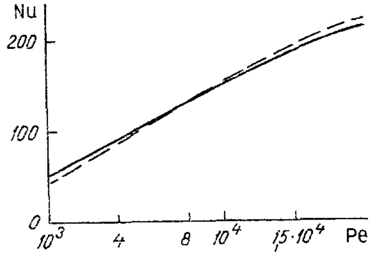

Fig. 1. Comparison of the theoretical solution (15) at Pr = 0.0053, $Pr_t = 1$, m = 12, n = 7, $\gamma = 0.0092$ with empirical solution obtained by E. D. Fedorovich Nu = 0.46 $Pe^{0.65}$ (dashed curve).

Here, in accordance with the experimental data in [1, 3-5], the velocity profile of the turbulent TBL core (for large values of η) is approximated by the function

$$W_2(\eta) = \eta_1 + B(\eta - \eta_1)^{\frac{1}{m}}, \tag{10}$$

where the value of the constant B is determined from the boundary condition on the external boundary of the hydrodynamic TBL.

We find the temperature gradient in the region $[k_1, k_2]$ from the solution of Eq. (2) with the value of ε_s found from (8):

$$\Delta t_2 = \frac{Q}{W_k}\,\frac{\Pr C(k_1,\ k_2)}{2\sqrt{2}z}, \tag{11}$$

where

$$C(k_1,\ k_2) = \frac{1}{2}\ln\frac{(p^2 + p\sqrt{2} + 1)(g^2 - g\sqrt{2} + 1)}{(p^2 - p\sqrt{2} + 1)(g^2 + g\sqrt{2} + 1)} +$$
$$+ \operatorname{arctg}\frac{\sqrt{2}zk_2(1 - g^2) - k_1(1 - p^2)}{(1 - p^2)(1 - g^2) + 2pg};$$
$$p = zk_2; \quad g = zk_1; \quad z = \varkappa\sqrt[4]{\frac{\gamma \Pr}{\Pr_t}}.$$

To determine heat transfer in the turbulent core of the TBL, we assume that considerably more energy is transferred by eddy diffusion than by molecular diffusion. In this case, we exclude ν and $\nu/\Pr$ from Eqs. (1)-(2). These equations then allow the particular solution [6]

$$dt = \frac{Q \Pr_t}{W_k}dW(\eta). \tag{12}$$

After integration of (12), the temperature gradient in the turbulent core of the TBL has the form

$$\Delta t_3 = \frac{Q}{W_k}\Pr_t\left[\sqrt{\frac{2}{c_f}} - W_2(k_2)\right]. \tag{13}$$

The complete temperature gradient in the TBL is the sum of Eqs. (7), (11), and (13), i.e.,

$$\Delta t = \frac{Q}{W_k}\left\{A\eta_1 \Pr^{\frac{2}{3}} + \frac{\Pr C(k_1,\ k_2)}{2\sqrt{2}z} + \Pr_t\left[\sqrt{\frac{2}{c_f}} - W_2(k_2)\right]\right\}. \tag{14}$$

After we insert the heat-transfer coefficient, Eq. (14) appears as follows in dimensionless similarity criteria

$$\mathrm{Nu} = \frac{z\sqrt{0{,}5c_f}\,\mathrm{Pe}}{z\left\{\Pr_t\left[\sqrt{\frac{2}{c_f}} - W_2(k_2)\right] + A\eta_1\Pr^{\frac{2}{3}}\right\} + \frac{\Pr C(k_1,\ k_2)}{2\sqrt{2}}}. \tag{15}$$

With Pr = 1, $k_1 = k_2 = \eta_1$, and Eq. (15) yields the well-known [1, 3-5] Reynolds analogy of the coefficient of heat transfer between a plate and a fluid in the absence of a pressure gradient in the external flow:

$$\mathrm{St} = 0{,}5c_f.$$

Figure 1 shows the agreement between the theoretical solution (15) and the experimental data. The slight difference is due to the following: 1) neglect by the model of the heat generated in the friction of the heat-transmitting flow of liquid metal on the plate; 2) the assumption of passivity ($Pr_t = 1$) of the physical quantities being transferred (momentum and heat); 3) neglect of the effect on heat transfer of the additional phase (impurities, oxides) formed at the interface of the liquid metal and wall, as well as the effect of free convection in liquid metals.

The solution (15) can also be used for approximate calculation of heat exchange between a hydraulically smooth plate and a flow of an incompressible gas ($Pr \le 1$).

NOTATION

ν, kinematic viscosity, m^2/sec; ε_s, hydrodynamic coefficient of eddy diffusion, m^2/sec; $W(\eta) = u_x(y)/W_k$, dimensionless velocity; $u_x(y)$, projection of averaged velocity on the x axis, m/sec; $W_k = \sqrt{\tau_c/\rho}$, "dynamic" velocity, m/sec; τ_c shear stress on the surface of the plate, N/m^2; ρ, density of the fluid, kg/m^3; $\eta = yW_k/\nu$, dimensionless coordinate; $Q = q/(\rho c_p)$; q, heat flux, W/m^2; c_p, heat capacity, J/(kg·deg); $Pr_t = \varepsilon_s/\varepsilon_q$, turbulent Prandtl number; ε_q, temperature coefficient of eddy diffusion, m^2/sec; $Pr = \nu/a$, molecular Prandtl number; a, diffusivity, m^2/sec; t, temperature, K; $c_f = 0.074/Re^{0.2}$, plate friction coefficient; κ, dimensionless turbulence coefficient; γ, universal constant equal to 0.0092 after Dugdale or 0.0125 after Hanratty [1]; $Nu = \alpha L/\lambda$, Nusselt number, α, heat-transfer coefficient, $W/(m^2 \cdot K)$; λ, thermal conductivity, $W/(m \cdot K)$; L, plate length, m; Pe = RePr, Peclet number; $Re = u_\infty L/\nu$, Reynolds number; St = Nu/Pe, Stanton number.

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